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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01

Paper 1

General comments

The overall standard was high both with regard to presentation and to the quality of the essential mathematics. Most candidates attempted all the questions, and there were few misreads. There were some elementary errors to be seen, but these, in most cases, did not seriously depress script totals.

Candidates generally showed high levels of knowledge and expertise with such syllabus topics as summation of series, determination of asymptotes of curves defined by a rational function, reduction formulae, mean values, second derivative determination and evaluation of area of a surface of revolution in the context of parametric representation, and also solution of standard second order differential equations. In contrast, induction, complex numbers and linear algebra were clearly not well understood.

There was a wide range of distribution of marks. At the lower end, there were a few script totals which indicated serious lack of preparation for this examination whereas, at the other extreme, a very high standard was attained by a significant minority of candidates.

Comments on specific questions

Question 1

Generally, this introductory question was answered accurately. Some of the methods adopted were sub-optimal with respect to use of examination time.

Most candidates began by writing $S_N = 8\left(\frac{1}{4}\right)N^2(N + 1)^2 - 6\left(\frac{1}{6}\right)N(N + 1)(2N + 1)$, and then showed about the right amount of working to establish the required result. There were a few attempts to use induction, but whether correct, or not, these could not be awarded any credit, for the question specifically demands use of standard results in the List of Formulae.

In the second part of the question, not all candidates recognised that the required sum is $S_{2N} - S_N$ but instead worked from incorrect forms such as $S_{2N} - S_{N+1}$, or $S_{2N} - S_{N-1}$.

Moreover the correct $\sum_{n=N+1}^{2N}\left(8n^3 - 6n^2\right) = 2N(2N+1)\left(8N^2 - 1\right) - N(N+1)\left(2N^2 - 1\right)$ was not always accurately transformed to the displayed result.

Answer: $N\left(30N^3 + 14N^2 - 3N - 1\right)$.

Question 2

The majority of candidates obtained correct results for part **(i)**. Responses to part **(ii)**, however, were generally incomplete and/or erroneous.

(i) Almost all responses stated, correctly, that the vertical asymptote is $x = 1$. Beyond that, there generally appeared some attempt to obtain the intermediate result

$$y = -ax + 1 - a + (1 - a)/(x - 1) \ (*)$$

from which the oblique asymptote can be obtained immediately. Actually a substantial number of such division processes were erroneous but, all the same, led to the required equation. Also, a small minority of divisions were incomplete and these, more often than not, led to a result such as $y = -ax$.

In passing, it must be remarked that the strategy based on the obtaining of (*) is sub-optimal, for the oblique asymptote is clearly of the form $y = -ax + c$ so that $x - ax^2 \approx -ax(x - 1) + c(x - 1)$ for large x. Thus, equating coefficients of x leads at once to $c = 1 - a$. However, a small minority of candidates argued successfully in this way.

In contrast, a few employed the classical method by setting $mx + c = \dfrac{x - ax^2}{x - 1}$ to obtain $(m + a)x^2 + (c - m - 1)x - c = 0$. Tangency at infinity implies $m + a = 0$, $c - m - 1 = 0$ and hence $m = -a$, $c = 1 - a$.

(ii) Most candidates differentiated the given function of x, set the result equal to zero and then eventually obtained $x = \dfrac{2a \pm \sqrt{4a^2 - 4a}}{2a}$.

Few differentiated the form (*) in part **(i)** so as to obtain the more useful result $x = 1 \pm \sqrt{1 - \dfrac{1}{a}}$ for the x-coordinates of the turning points of C. To prove that both of these values, x_1, x_2, say, are positive, an argument such as the following was expected.

$a > 1 \Rightarrow x_1$ and x_2 are real. Also, $a > 1 \Rightarrow 1 - \dfrac{1}{a} < 1 \Rightarrow x_1 > 0$ and $x_2 > 0$.

However only about half of all candidates succeeded in proving that x_1, x_2, are real and only a minority could go on to show that they are positive.

Answer: **(i)** Asymptotes are $x = 1$ and $y = -ax + 1 - a$.

Question 3

Most candidates made some progress here, but there were few outstanding responses.

(i) Almost all candidates correctly transformed the given $x - y$ equation into the polar form. There were few instances of omission of essential detail.

(ii) Most responses showed, at least, a partially correct sketch for $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$. However, in many cases there appeared either no other loop or, at the other extreme, loops for $\dfrac{\pi}{2} \leqslant \theta \leqslant \pi$ and/or $\dfrac{3\pi}{2} \leqslant \theta \leqslant 2\pi$ where, in fact, C does not exist. Thus those responses which showed correct loops in $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$ and $\pi \leqslant \theta \leqslant \dfrac{3\pi}{2}$, and no other loops, were very much in a minority.

(iii) Most responses showed an intelligent use of $r^2 = 2\sin 2\theta$ to obtain the maximum of r. There were few incorrect answers.

Question 4

Most candidates found this question beyond them and so there were very few complete and correct responses.

(i) At this level it was to be expected that the successive differentiation of $\frac{\ln x}{x}$ with respect to x would be a routine task for candidates. Nevertheless, a significant number of elementary errors appeared and this indicated a deficiency in basic mathematical technique in the candidature. Beyond that, some candidates did not even comprehend that three differentiations were required in order to establish the values of a_1, a_2, a_3, and this lack of perception inevitably led to an incorrect conjecture for the form of a_n, as required in the remainder of this question.

(ii) A minority of candidates wrote down a correct inductive hypothesis. Among those who did work from $H_k : a_k = (-1)^k k!$, few went on to prove convincingly that $H_k \Rightarrow a_{k+1} = (-1)^{k+1}(k + 1)!$ and hence to complete the inductive argument. Again, it was evident that lack of technique was the main cause of failure.

Answers: **(i)** $a_1 = -1$, $a_2 = 2$, $a_3 = -6$; **(ii)** $a_n = (1)^n n!$

Question 5

This question enjoyed only moderately success attempts. Many candidates became involved in unnecessarily protracted arguments.

In most cases, the correct eigenvalues appeared from the working. Nevertheless, a significant minority of candidates did not understand that as the matrix **A** is triangular, then the diagonal elements are the eigenvalues, In consequence, they became involved in the labour of deriving and solving the characteristic equation of **A** and this exercise must have wasted a lot of examination time.

Most candidates employed sound methodology for the obtaining of the eigenvectors, but a profusion of elementary errors led to many failures in this respect. In particular, it was not uncommon for $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ to be obtained as the eigenvector corresponding to the eigenvalue 1

In the rest of the question, most candidates produced results which were consistent with the results previously obtained. Very few wasted time by attempting to determine $\mathbf{P}^{-1}$.

Answers: The eigenvalues of **A** are 1, 3, 4: corresponding eigenvectors are: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}$.

$\mathbf{D} = \text{diag}(1, 243, 1024)$, $\mathbf{P} = \begin{pmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & -3 \end{pmatrix}$.

Question 6

This question was well answered by the majority.

The most popular strategy for establishing the displayed reduction formula was by the method indicated in the question. In this respect, working was generally accurate and complete. In contrast, a minority of candidates integrated $(\ln x)^n$ by parts to obtain I_n in terms of I_{n-1} and then upgraded n to $n + 1$. This method was also applied correctly, in most cases.

For the final part, working was generally relevant, accurate and complete.

Question 7

This question is a straightforward exercise in complex numbers requiring only a basic knowledge of the related syllabus material. Nevertheless the majority of candidates appeared not to understand the essence and not to have the necessary technical expertise for answering questions of this type.

Most candidates did, at least, get as far as obtaining a correct result for $|z|$. In contrast, candidates generally produced three possible values for arg z, but it was common for at least one of them not to be within the stipulated range of θ, or to be completely wrong.

The key to the remaining part of this question is to observe that if z_1, say, is any root of the complex equation $z^3 = -4\sqrt{3} + 4\text{i}$, then $z_1^{3k} = \left[z_1^3\right]^k = \left(-4\sqrt{3}+4\text{i}\right)^k = \left[2^3 e^{5\pi\text{i}/6}\right]^k = 2^{3k}.e^{5\pi k\text{i}/6}$.

The displayed result is then immediate. However, a minority of candidates could produce an equivalent argument.

Answer: $z = 2\exp\left(\frac{5\pi\text{i}}{18}\right)$, $2\exp\left(\frac{17\pi\text{i}}{18}\right)$, $2\exp\left(\frac{29\pi\text{i}}{18}\right)$.

Question 8

There were many responses to this question which developed along the right lines, but few of these could be described as outstanding.

(i) As is so often the case with questions of this type, almost all candidates produced a correct result for $\frac{\text{d}y}{\text{d}x}$ in terms of t, but then attempted to go on to obtain the second derivative on the basis of erroneous formulae such as $\frac{\text{d}^2y}{\text{d}x^2} = \frac{\text{d}}{\text{d}t}\left(\frac{\text{d}y}{\text{d}x}\right)$ and $\frac{\text{d}^2y}{\text{d}x^2} = \frac{\frac{\text{d}^2y}{\text{d}t^2}}{\frac{\text{d}^2x}{\text{d}t^2}}$, the former being much more prevalent than the latter. In cases where $\frac{\text{d}^2y}{\text{d}x^2}$ was correctly obtained in terms of t, most candidates expressed their result in the form $\frac{-2\left(1+t^2\right)}{3\left(t^2-1\right)^3}$ and supplemented this with a valid and complete argument to prove that $\frac{\text{d}^2y}{\text{d}x^2} < 0$ at all points of C.

(ii) Most responses started correctly with $S = 2\pi\int_2^3\left(3t^2+1\right)\left[\left(3t^2-3\right)^2+36t^2\right]^{1/2}\text{d}t$ as the integral representation of the required surface area. For the evaluation, most candidates perceived that use of the identity $[(3t^2 - 3)^2 + 36t^2]^{1/2} = 3t^2 + 3$ is essential and went on to work accurately towards the final, numerical result for S.

A small minority evaluated the corresponding arc length instead of S, and this might suggest a lack of understanding of the terminology of the related syllabus material.

Answer: **(ii)** 2880.

Question 9

This question generated a lot of good work, but there were relatively few completely correct responses. In general, candidates did best with the last part of the question.

In the introductory part, almost all responses showed a correct argument to establish the preliminary result $\frac{\mathrm{d}y}{\mathrm{d}x} = -t^2\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)$. In contrast, less than half of all candidates could prove the associated second order result. Many understood that it could, in principle, be derived from $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left[-t^2\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\right]$, but lacked the necessary insight to continue further. In fact, a candidate needed to provide the following:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right]\frac{\mathrm{d}t}{\mathrm{d}x} = \left[-t^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right) - 2t\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\right](-t^2) = t^4\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right) + 2t^3\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)$$, or equivalent.

The middle section requires no more than substitution of the results previously obtained plus accurate simplification and in this context there were few failures.

In the final part, most responses showed a sound strategy. The correct complementary function usually appeared, but there were a surprisingly large number of errors to be seen in the working for the particular integral.

Answer: General solution: $y = Ae^{-1/x} + Be^{-4/x} + \frac{2}{x} + 1$.

Question 10

This question highlighted much erroneous thinking with regard to the fundamentals of linear spaces and systems of linear equations.

(i) Many responses lacked clarity and frequently the impression was given that the candidate did not understand what was required. In fact, a, simple way to proceed is to obtain an echelon form, such as $\begin{pmatrix} 2 & -1 & 4 & -5 \\ 0 & 5 & -6 & 11 \\ 0 & 0 & 0 & \theta+6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. It is then a simple matter to deal with the cases $\theta \neq -6$ and $\theta = -6$. In this respect what was required were arguments such as $\theta \neq -6 \Rightarrow \dim(K) = 4 - \mathrm{r}(\mathbf{A}) = 4 - 3 = 1$ and $\theta = -6 \Rightarrow \dim(K) = 4 - \mathrm{r}(\mathbf{A}) = 4 - 2 = 2$. However, few candidates supplied such detail.

(ii) For the most part, candidates' working was accurate.

(iii) A standard procedure here is to express x, y, z, t in terms of exactly two parameters, e.g., $x = -7\phi + 7\psi$, $y = 6\phi - 11\psi$, $z = 5\phi$, $t = 5\psi$.

This not only provides a check of the result for $\mathbf{e}_1$ obtained in part **(i)**, but also enables a result for $\mathbf{e}_2$ to be written down.

(iv) In general terms, it was expected that the candidate would verify clearly that $\mathbf{A}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 10 \\ 15 \end{pmatrix} = \mathbf{b}$, and then set out working such as, $\mathbf{Ax}_1 = \mathbf{Ae}_o + k_1\mathbf{Ae}_1 + k_2\mathbf{Ae}_2 = \mathbf{b} + k_1\mathbf{0} + k_2\mathbf{0} = \mathbf{b}$, for all k_1, k_2.

Some responses appeared to proceed along these lines though, it must be said, many such arguments were incomplete in some important way.

Answers: **(ii)** $\mathbf{e}_1 = \begin{pmatrix} 7 \\ -6 \\ -5 \\ 0 \end{pmatrix}$; **(iii)** $\mathbf{e}_2 = \begin{pmatrix} 7 \\ -11 \\ 0 \\ 5 \end{pmatrix}$.

Question 11 EITHER

Of those who attempted this question, about half worked through the first three sections without error. However, only a small minority made any progress in part **(iv)**.

(i) Almost all responses showed accurate and relevant working. A few, however, gave the complementary angle as the answer.

(ii) The relevance of the vector product (**i** – **k**) x (2**i** + 2**j** + **k**) was perceived by most and usually, though not always, this was evaluated accurately.

(iii) Some candidates made no progress beyond the end of part **(ii)**. They failed to realise that the direction of the line of $\Pi_1 \cap \Pi_2$ is parallel to (**i** – **k**) x (2**i** – 3**j** + 2**k**) and this failure would suggest lack of geometrical insight. In this respect, they would probably have made much better progress had they drawn an effective diagram in the first place. This would also have made clear to them that the three vectors **i** – **k**, 2**i** + 2**j** + **k**, 3**i** + 4**j** + 3**k** are parallel to the plane Π_2 and are therefore linearly dependent. As it was, hardly any candidates argued in this simple way, but instead employed extended algebraic methods.

(iv) A simple strategy here begins with the specification $Q \equiv (u, v, u)$. Thus since $l \perp m$, then $2(u-4) + 2(v-4) + (u-2) = 0 \Rightarrow 3u + 2v - 18 = 0 \Rightarrow OQ^2 = 2u^2 + \left(9 - \frac{3u}{2}\right)^2$. The minimum of OQ^2 and hence of OQ can then easily be found.

Answers: **(i)** 13.6°; **(ii)** $2x - 3y + 2z = 0$; **(iii)** **r** = t(3**i** + 4**j** + 3**k**); **(iv)** 6.17.

Question 11 OR

The quality of responses to this question was better than that for **Question 11 EITHER**. However, elementary errors downgraded a substantial amount of the work submitted.

(i) There were very few failures at this stage.

(ii) Some candidates evaluated S_4 from the y-equation, but thought they had evaluated S_2. Yet others attempted to use $S_4 - S_2 - S_1 = 0$, and were generally successful.

(iii) From this stage onwards many responses ran into confusion and were frequently incomplete, or simply wrong. A popular but uninspired method was to generalise the previous methodology by writing $S_{n+3} - S_{n+1} - S_n = 0$, and then choose suitable values of n. This enjoyed some success, but even so, a more expeditious strategy is to set $y = \sqrt{t}$ and so obtain $t^3 - 2t^2 - 3t - 1 = 0$ in a very similar way to the derivation of the y-equation from the given x-equation. As the t-equation has roots α^4, β^4, γ^4 then the sums of the squares, cubes and fourth powers of this equation are S_8, S_{12}, S_{16}, respectively, and so following the earlier methodology these may be obtained without difficulty.

Answers: **(ii)** $S_4 = 2$; **(iii)** $S_8 = 10$, $S_{12} = 29$, $S_{16} = 90$.

Paper 9231/02

Paper 2

General comments

As is usually the case, candidates seemed on the whole to be more at ease with the Statistics questions. This was particularly true in the single question offering alternative choices, namely **Question 12**, where virtually all candidates elected to attempt the Statistics option. Despite this general preference, most candidates had sufficient time to attempt all the questions on the paper, indicating that the time pressure was not unduly great.

As indicated in more detail below, there was considerable variation between questions in the level of success, with almost no candidates offering completely correct answers to **Questions 1** or **6**, for example. By contrast, others such as **Questions 3** and **10** often produced good work.

Comments on specific questions

Question 1

The great majority of candidates appreciated that the change in momentum, found from the product of the mass and the change in velocity, should be divided by the time of contact in order to give the force causing the change, though some overlooked the fact that the two given speeds are in opposite directions and therefore should be added rather than subtracted. Only very rarely, however, did a candidate realise that the weight of the hammer contributes to the force on the nail.

Answer: 1530 N.

Question 2

This is clearly a question on angular motion, but a large number of candidates used a variety of stratagems, almost always invalid, to transform the question into one on linear motion. Instead the product of the flywheel's moment of inertia and angular deceleration should be equated to the couple acting on it, since this constant deceleration acting for the required time must equal the given initial angular speed. Some of those candidates who appreciated this general principle were unfortunately rather cavalier with the distinction between acceleration and deceleration. The loss in energy equals the initial rotational energy, found from one half the product of the moment of inertia and the square of the initial angular velocity, in appropriate units.

Answers: 25 s, 625 kJ.

Question 3

In each of the two collisions the resulting velocities are readily found by formulating and then solving the conservation of momentum and the restitution equations. Where errors occurred, they were more often due to faulty arithmetic than confusion over signs, since the motion of all spheres is in the same direction. This fact together with the final speeds means that it is immediately clear that there can be no further impacts after the second collision.

Answers: At rest; 1 ms^{-1}; 2 ms^{-1}.

Question 4

The easiest way of finding the vertical reactions at *A* and *B* is to formulate one equation, moments for the system about *B* for example, which yields one reaction, and then a second equation for the other, such as moments about *A* or vertical resolution of forces for the system. Many candidates needlessly complicated matters, though, by introducing unknown forces such as the friction at *A* and *B*, or the forces between the ladders at *C*. The latter may be readily found by, for example, resolving vertically for one of the ladders, and taking moments about the opposite end from *C* for either ladder. The magnitude of the resultant and its direction may then be found in the usual way, though it is necessary to specify the latter explicitly rather than simply calculating an undefined angle.

Answers: $\frac{5W}{4}$, $\frac{3W}{4}$; $\frac{1}{2}W$, 30° above horizontal.

Question 5

The first part requires the formulation of a conservation of energy equation relating to the point *B*, involving cos *ACB*. It is also necessary to resolve forces in the radial direction, noting that the cylinder exerts no force on the particle as the latter leaves the surface at *B*. Combination of these two equations yields the given speed and the value of cos *ACB*. While the Examiners thought it preferable to deduce cos *ACB* and the speed in this way, they awarded full credit to those candidates who demonstrated instead that the given values satisfy the two equations. There are a variety of ways of finding the horizontal distance which the particle moves between leaving the surface and striking the table. Finding the time taken by considering the vertical component of the motion, or using the standard trajectory equation, both result in a quadratic equation for the time and horizontal distance respectively, while finding the vertical speed at impact avoids the need to solve a quadratic. The horizontal motion is of course under no acceleration. The distance *CD* requires the addition of the horizontal distance of *B* from *AC*.

Answer: 1.20*a*.

Question 6

Although this question was almost always attempted, completely correct answers were rare. Some candidates found the cumulative distribution function of X, namely $F(x) = x - 1$, and then simply replaced x by $\frac{2}{y}$ in an invalid attempt to find the cumulative distribution function $G(y)$ of y, giving an incorrect probability density function $g(y) = -\frac{2}{y^2}$. Others used the more valid approach of substituting $\frac{2}{X}$ for Y in $P(Y < y)$, which should yield $P(X > \frac{2}{y})$ and hence $G(y) = 1 - F\left(\frac{2}{y}\right) = 2 - \frac{2}{y}$, but they mishandled the inequalities and instead obtained $P(X < \frac{2}{y})$.

Answer: $\frac{2}{y^2}$.

Question 7

Most candidates readily found the value 3 of E(N), but a significant number miscalculated the probability in the first part of the question, taking it to be $P(N \leqslant E(N)) - P(N \leqslant 1)$, for example. The second part seemed more challenging, but simply requires the summation of an infinite geometric series with first term $\frac{1}{3}$ and ratio equal to the product of $\frac{2}{3}$ and $\frac{3}{4}$.

Answers: $\frac{19}{27}$, $\frac{2}{3}$.

Question 8

(i) The required probability is essentially found by calculating $1 - F(360)$, where $F(x)$ is the cumulative distribution function $1 - e^{-x/120}$, but many candidates used instead a broadly similar expression involving e^{-120x}.

(ii) Correct answers were very rarely seen, with many candidates taking P_n to be $F(n)$ or something similar, rather than the correct $F(n) - F(n - 1)$. The latter leads to a geometric progression with ratio equal to $e^{-1/120}$.

Answer: **(i)** e^{-3}.

Question 9

The principal potential stumbling block is obtaining an unbiased estimate of the population variance, which should be found from $\frac{8}{7}\left(\frac{11.2}{8} - (m - 30)^2\right)$, where m is the sample mean. The confidence interval may then be found in the usual way, using the tabular t-value 2.365.

Answer: [28.4, 29.9].

Question 10

Calculation of the expected values of the number of batches requires the correct probability of a tulip being yellow, namely 0.35, though other values were seen quite often. Another frequent error was a failure to combine the first two cells as well as the last four ones. The calculated χ^2 value 1.4 is then compared with the tabular value 7.815 in order to conclude that the binomial distribution fits the data.

Question 11

The two product moment correlation coefficients (pmcc) were usually determined by a valid method, though arithmetical errors were sometimes committed. Since the value 0.805 for sample *A* is greater than the corresponding value 0.719 for *B*, the regression line must be found for the former, and *c* taken to be 12 in order to predict *e*. Although most candidates stated that the pmcc of *C* is the same as that of *B*, only a minority gave a sufficient justification, which hinges on the fact that the square of the pmcc equals the product of the gradients of the two regression lines, and these gradients are of course equal for *B* and *C*. The pmcc for the combined sample is found in the same way as those for *A* and *B*, and its value 0.768 is compared with the tabular value 0.707 to deduce that the population pmcc is not different from 0.

Answer: 67.2.

Question 12 EITHER

This was a very unpopular alternative, and not particularly well answered by the few candidates who chose it. Showing that the motion is simple harmonic involves as usual relating the acceleration of the stone about the centre of motion to the net force on it, arising from its weight and the force exerted by the compressed spring. The distance x_0 of the centre of motion below *T* is readily found by equating these two opposing forces, and hence the distance $0.6 - x_0$ of this point above the floor. Having determined the square of the speed *v* of the stone when it makes contact with the spring at *T* to be $2gh$, this may be inserted into the standard formula $v^2 = \omega^2 (a^2 - x^2)$ with *x* equal to x_0 in order to find the amplitude *a*, since the value of ω has already been found. Conservation of energy provides an alternative method. The stone will not hit the floor provided *a* is less than $0.6 - x_0$, yielding the given inequality for *h*.

Answers: 0.36 m, $\sqrt{(0{\cdot}24^2 + 0{\cdot}48h)}$.

Question 12 OR

Most of the many candidates who attempted this question used the correct approach to determining the confidence interval, though not all chose the correct tabular *t*-value of 2.447, and some estimated the population variance inaccurately by using a rounded value of the sample mean with too few significant figures. A frequent error when estimating the common variance from the standard pooled formula was to confuse biased and unbiased estimates when inserting the previously calculated estimates of the two population variances. The final test presented few difficulties, with comparison of the calculated value 1.58 of *t* with the tabular value 1.833 leading to the conclusion that accidents do not occur more frequently in the town than in the country.

Answers: [6.02, 11.12], $\frac{607}{63}$.